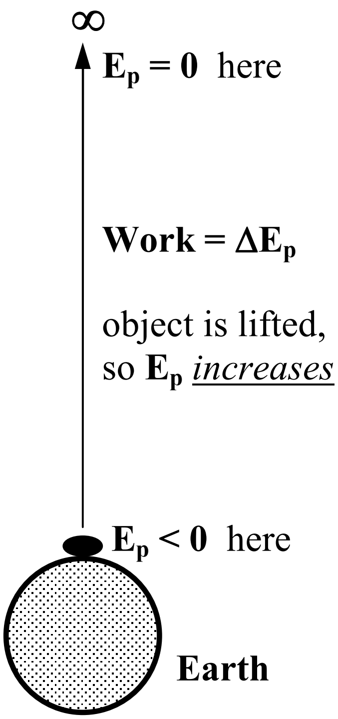
Gravitational Potential Energy in Space

When we are near the Earth's surface, we calculate the ΔE_p of an object as the **work done** to lift the object to its new height, where $E_p = mgh$ near the Earth's surface. The value of "**h**" is measured relative to some point that is chosen by you to be where the object will not fall any further. For example:

- For a pendulum, $h = 0$ at the bottom of its swing.
- For an object that is dropped, $h = 0$ where the object lands (usually on the ground).

However, because **g** changes with altitude, we can't use this formula in space. In space, if we move one mass away from another far enough (so that $R = \infty$), we will move it out of that mass's field and so E_p will finally equal zero.



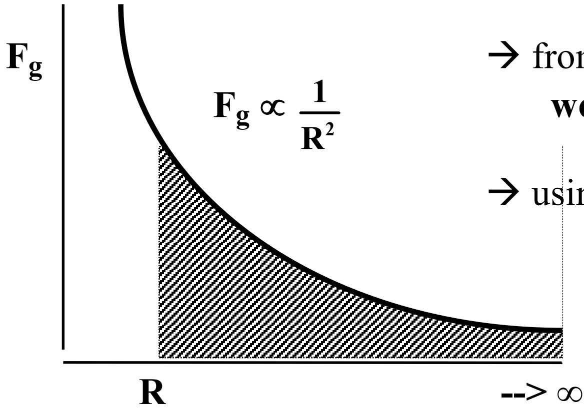
Whenever an object is lifted, potential energy is increased, resulting in *positive work done.*

If that object is lifted to *infinity*, where $E_p = 0$, then the potential energy of the object must be *less than zero* at any height that is closer than infinity; this includes the earth's surface.

Put another way, when an object falls, E_p decreases. Since the E_p decreases as the object falls from infinity to Earth, and since $E_p = 0$ *at* $R = \infty$, the E_p becomes increasingly *negative*.

We need to find an expression for E_p of an object at the surface of the Earth, compared with $E_p = 0$ at $R = \infty$.

From graphical analysis of $\mathbf{F_g}$ vs. $\mathbf{R}$: (remember that $\mathbf{F_g} = \frac{\mathbf{GMm}}{\mathbf{R^2}}$)



→ from any point $\mathbf{R}$ out to ∞,
work done = area under the graph

→ using calculus, $\mathbf{W} = \Delta \frac{\mathbf{GMm}}{\mathbf{R}}$

- but $\mathbf{W} = \Delta \mathbf{E_p} = \mathbf{E_{p\,(f)}} - \mathbf{E_{p\,(i)}} = \mathbf{0} - \mathbf{E_{p\,(i)}}$
- $\mathbf{E_p}$ is negative for <u>any value</u> of $\mathbf{R} < \infty$; that is, $\boxed{\mathbf{E_p} = -\frac{\mathbf{GMm}}{\mathbf{R}}}$

If this makes no sense to you, don't panic! The above is only a short, non-calculus explanation for the new formula for $\mathbf{E_p}$. What you <u>must</u> know is this: the formula $\mathbf{E_p} = -\frac{\mathbf{GMm}}{\mathbf{R}}$ is used to find potential energy for an object '**m**' in space relative to:

- the central planetary mass '**M**'.
- $\mathbf{E_p} = \mathbf{0}$ at infinity.

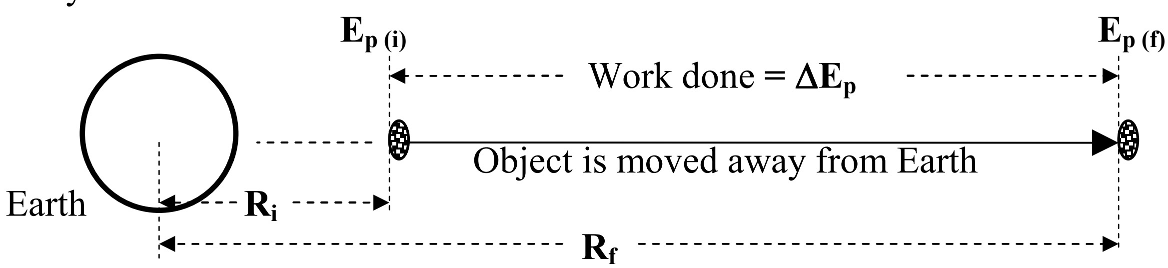
Example 9: Find the potential energy (relative to infinity) of a 50. kg person flying at an altitude of 1.0 x 10^4 m above Earth's surface, relative to infinity.

(see Gravitation Ex 9 for answer)

Example 10: If that 50. kg person was in a space shuttle orbiting at an altitude of 250 km, what would be her new potential energy, relative to infinity?

(see Gravitation Ex 10 for answer)

Now examine the situation below where work is done to move an object further away from Earth:



Recall that work is equal to change in energy. In this example, the **work done** = the difference between the initial E_p and the final E_p.

$$W = \Delta E_p = E_{p\,(f)} - E_{p\,(i)} = -\frac{GMm}{R_f} - \left(-\frac{GMm}{R_i}\right)$$

$$W = GMm\left(\frac{1}{R_i} - \frac{1}{R_f}\right)$$ → energy added is *positive* energy.

Example 11: Determine the work done to move a 3.5×10^4 kg cargo of space junk from an altitude of 430 km above the Moon's surface to a radial distance of 2.8×10^6 m away.

(see Gravitation Ex 11 for answer)